MECHANICAL MODELING OF GLASS AND CARBON EPOXY COMPOSITES

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ABSTRACT

A Theoretical model to predict the response of laminated composites is developed. The micromechanical model simulates the mechanical response of a multi layer composite laminate under uniaxial, biaxial, and flexural loading modes. The stacking sequence is utilized to obtain the overall stiffness matrix for each lamina. Effect of distributed cracking on the stiffness degradation of the cross ply layers under tensile loading is measured using a scalar damage parameter. The model is calibrated by predicting the ultimate strength of unidirectional, cross ply, and angle ply laminates under tensile and flexural loading.

INTRODUCTION

In order to commercially utilize new composite materials in civil engineering applications, simple and effective analysis and design guides are needed. Theoretical models are also needed to predict the response of laminated composites in order to better understand the interaction between the various phases and aid in the design of overall structural system. The present work presents a general framework for analysis and design for modeling the uniaxial and flexural response of composite laminates. The proposed methodology can be used as a new composite material or used as retrofitting and strengthening component of an existing structure.

Each single lamina is modeled as an orthotropic sheet in plane stress. Depending on the state of strain (normal and shear) and curvature distribution, strain at the top and bottom of each lamina is calculated. The strain distribution is applied to the orthotropic model to calculate the stress in each ply. The degradation of the stiffness properties is considered using a strain based scalar damage-softening model. Two zones of behavior are considered for the matrix phase and include the elastic response, and stiffness degradation due to initiation and generation of parallel crack formation. The modeling steps are categorized as follows:

Elastic matrix- The rule of mixtures and the Halpin-Tsai [1967] estimates of transverse modulus are applicable. This zone is terminated by the failure of the matrix phase using a stress-based criterion [Agarwal and Broutman 1990].

Cracked-Softening Matrix-Within this range, as the strain is increased, the stress carried by the matrix is decreasing in terms of a strain-softening law. The form of the strain softening law proposed by Horii et al. [1987] was used. The load carrying capacity of the matrix phase in each lamina decreases and the response of the lamina degrades such that the composite response asymptotically approaches the levels predicted by the ply discount method. For a lamina with its matrix phase in unloading mode, a proportional unloading for the stresses in other directions was assumed.

In the present calculations, the experimental data of continuous glass, woven carbon composites are presented. Glass epoxy systems have been used in the composites area for many years and they present a wealth of data for comparison of with analytical models. In addition, the

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advancement of application of woven composites requires proper procedures for calibrating the response of a highly complex system to simple analytical techniques. There are several research programs where fabrics are being studied for potential structural applications. Generation and use a macroscopic material model in the numerical (finite element) simulation of fabrics is a challenging problem. Modeling of individual yarns as a part of the entire finite element model is not possible given today's computing capabilities. The approach is to develop an equivalent model, called the macroscopic model, whose performance is as close to the original fabric as possible. The state-of-the-art in fabric modeling may be attributed to the work done at NASA-Langley Research Center [B.N. Cox, G. Flanagan, 1997] and the TEXCAD computer program [Naik, 1994] to convert a woven fabric material model into the standard orthotropic properties. The present work addresses use of equivalent orthotrpic responses of these systems.

Equivalent Elastic Lamina Formulation

The equivalent elastic stiffness of the lamina is obtained using the sum of the contributions from each phase to the overall stiffness. The effect of fiber volume fraction is incorporated in the elastic properties. It is assumed that as microcracking in the composite takes place, the stiffness degrades according to a single scalar parameter 'ω'. The stiffness of the matrix phase is therefore defined as a function of damage and used in the rule of mixtures to obtain the stiffness of the lamina. The fiber was assumed to be linear elastic. Calculation of the transverse modulus E_2 and ν_{12} were achieved using the Halpin-Tsai equations:

$$E_1(\omega) = E_f V_f + E_m(\omega)(1 - V_f) \tag{1}$$

$$E_2(\omega) = \frac{E_m(\omega)(1 + \xi\, \eta V_f)}{1 - \eta V_f} \qquad \eta = \frac{E_f - E_m(\omega)}{E_f + \xi\, E_m(\omega)} \tag{2}$$

The value of ξ was taken as an adjustable parameter and set equal to 0.2 in the present study.

Matrix Degradation and Softening

There is a gradual decrease in the load carrying capacity of the matrix beyond the ultimate strength and is refereed to as the softening zone. A Model proposed by Hori was used to estimate the degradation of stress as a function of strain. The damage evolution was empirically based as:

$$\omega_i = \begin{cases} \omega_0 & \forall \quad f(\sigma_1) < 1 \\ \omega_0 + \alpha(\varepsilon_1 - \varepsilon_{um})^{\beta} \quad , & 0 \le \omega_i \le 1 \quad \forall\ f(\sigma_1) = 1 \end{cases} \tag{3}$$

In order to formulate the damage vs. strain relationship, the model proposed by Karihaloo and Fu, [1995] was used as shown in Equation 3. Parameters of α= 0.16, β= 2.3 were used, where as ε_{um} represents the ultimate strain at failure for uniaxial tension. The stress capacity is affected by the amount of damage reached at the peak, and the value of the stress and post peak deformation *w* are obtained from the damage parameter according to:

$$\frac{\sigma}{f'_t} = \sqrt{\frac{tan(\pi\omega_0 / 2)}{tan(\pi\omega / 2)}} \qquad \frac{w}{w_0} = \frac{\sigma}{f'_t}\left(\frac{log(sec\,\pi\omega / 2)}{log(sec\,\pi\omega_0 / 2)}\right) - 1 \tag{4}$$

$$\overline{E}_m = \frac{\overline{E}_m}{1 + \frac{16}{3}\omega(1 - \upsilon_m^2)}$$

where ω_0 is the damage accumulated at the peak stress. The value of ω_0 is obtained using the pre-peak response model from the magnitude of strain. The definition of strain in this region is gage length dependent and the present approach requires the definition of a mean strain over the length of several cracks in the matrix. The deformation at peak is w_0, and obtained as: $w_0 = \varepsilon_p H$, where H is the gage length of the specimen, and ε_p is the strain at peak stress. The sample in the strain-softening zone asymptotically approached a level of zero stress, comparable to the ply discount method, totally neglecting the stress in a cracked layer. As the specimen undergoes strain softening, a reduced stiffness was used for the section. The modulus E_m is computed for each strain level ε, using the relation stated in equation 4 and shown in figure 1 which expresses the reduction of stiffness due to increase of damage [Nemat Nasser and Hori, 1993].

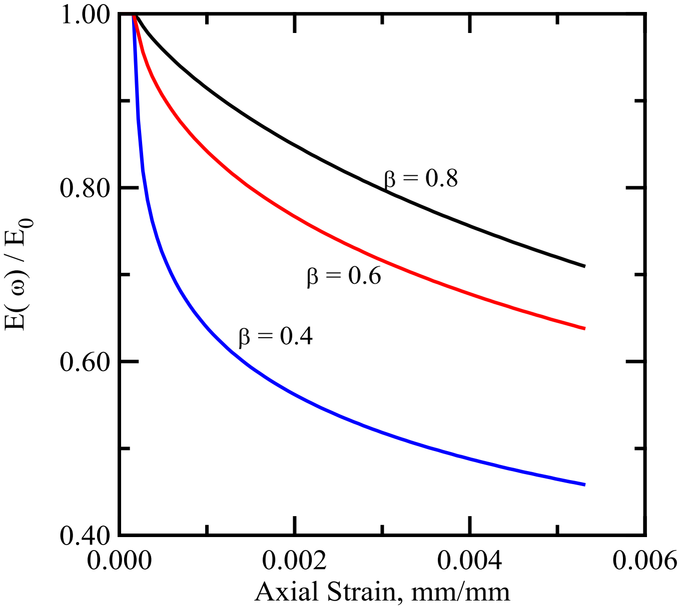

Figure 1. The stiffness degradation as a function of empirical parameter β.

Generalized Load-Displacement for the Composite Response

The constitutive relations for a general orthotropic material include the compliance matrix, *S*, or its inverse the stiffness matrix, *Q*, and relates the stress and strain within a lamina loaded in its principal directions [Jones, 1975]. Since the present model updates the elastic stiffness of the matrix due to cracking, an elastically equivalent compliance matrix $\overline{S}$ was defined where the bar indicates use of updated elastic properties. In the term S^i_{jk}, parameter "i" represents the load increment, "j" the direction of applied strain, and "k" the observed stress. The stress strain relationship was represented in incremental form for each loading increment i, as:

$$\begin{aligned}\Delta\varepsilon_j^i &= \overline{S}_{jk}^i \,\Delta\sigma_k \\ \sigma_k^i &= \left(\overline{S}_{jk}^i\right)^{-1} \Delta\varepsilon_j^i + \sigma_k^{i-1}\end{aligned} \tag{5}$$

Or in matrix form:

$$\sigma_k = \begin{bmatrix}\sigma_1 \\ \sigma_2 \\ \tau_{12}\end{bmatrix}_i = \begin{bmatrix}\overline{S}_{11} & \overline{S}_{12} & 0 \\ \overline{S}_{21} & \overline{S}_{22} & 0 \\ 0 & 0 & \overline{S}_{66}\end{bmatrix}^{-1} \begin{bmatrix}\Delta\varepsilon_1 \\ \Delta\varepsilon_2 \\ \Delta\gamma_{12}\end{bmatrix}_i + \begin{bmatrix}\sigma_1 \\ \sigma_2 \\ \tau_{12}\end{bmatrix}_{i-1} \tag{6}$$

$$\overline{S}_{11} = \frac{1}{E_{1(\omega)}} \qquad \overline{S}_{12} = -\frac{\upsilon_{12}}{E_{1(\omega)}} \qquad \overline{S}_{22} = \frac{1}{E_{2(\omega)}} \qquad \overline{S}_{66} = \frac{1}{G_{12(\omega)}} \tag{7}$$

By inverting the compliance matrix, S, the stiffness matrix, Q is obtained which relates the strains into stresses for each lamina loaded in principal material direction.

$$\overline{Q}_{ij}(\omega) = T_{ij}\, R\, \overline{S}_{ij}^{-1} R T_{ij}^{-1} \tag{8}$$

For a composite laminate consisting of several lamina each with an orientation of θ^m, where m represents the first to the nth ply, the classical lamination theory results in derivation of lamina stiffness components as:

$$\overline{A}_{ij} = \sum_{m=1}^{n} \overline{Q}_{ij}^{m} (h_m - h_{m-1}), \quad \overline{B}_{ij} = \frac{1}{2}\sum_{m=1}^{n} \overline{Q}_{ij}^{m} (h_m^2 - h_{m-1}^2), \quad \overline{D}_{ij} = \frac{1}{3}\sum_{m=1}^{n} \overline{Q}_{ij}^{m} (h_m^3 - h_{m-1}^3) \tag{9}$$

The form of submatrices A, B, and D is discussed by Agarwal and Broutman [1990], where A represents the extensional, D the bending, and B the coupling stiffness. With knowledge of strain and curvatures, the stress distribution per lamina is computed for each loading step in an incremental fashion. M represents the moment per unit length, N the force per unit length of cross section, ε^0 and κ represent the midplane axial strain and the curvature of the section respectively. The strains and forces were updated incrementally according to the matrix form representation:

$$\begin{bmatrix}\Delta N \\ \Delta M\end{bmatrix} = \begin{bmatrix}\overline{A}(Q(\omega)) & \overline{B}(Q(\omega)) \\ \overline{B}(Q(\omega)) & \overline{D}(Q(\omega))\end{bmatrix} \begin{bmatrix}\Delta\varepsilon^0 \\ \Delta\kappa\end{bmatrix} \tag{10}$$

After each iteration, the incremental loads and strains are determined and the results are added to the loads and strains at the previous ply failure. The applied load in the x direction at the ith interval in the jth lamina was represented as $N^{j}_{x,i}$ according to:

$$N_{x,i}^{j} = N_{x,i-1}^{j} + \Delta N_{x,i}^{j} = N_{x,i-1}^{j} + \left[\overline{A}(Q(\omega))\right]_i \left\lfloor \Delta\varepsilon^0 \right\rfloor \tag{11}$$

Failure Criteria

It is known that matrix in the 0 degree plies may be subjected to significant parallel microcracking due to the bridging effect of fibers. The matrix phase in the 90 degree plies loaded in tension may also be subjected to parallel cracking due to the shear lag of adjacent layers. For an off-axis lamina subjected to shear, the matrix phase may fail due to the formation of a single shear crack. The failure criterion for the first cracking of matrix was based on the state of stress and represented as the yield surface, F represented as:

$$F(\sigma_1,\sigma_2,\tau_{12}) = 1 \qquad \sigma_1 \geq \sigma_1^{fu} \qquad \sigma_2 \geq \sigma_2^{fu} \qquad \tau_{12} \geq \tau_{12}^{fu} \tag{12}$$

After each incremental loading, stresses in the lamina were checked against the failure surface to update the material properties for subsequent analysis. For a unidirectional lamina subjected to tension, assuming that it is sufficiently loaded such that the matrix phase has cracked significantly, the ultimate tensile strength was set equal to the strength of the fiber phase, and represented as:

$$\sigma^{ut}(\theta) = \max\ (V_f\ \sigma^{fu}\ Cos^2\theta,\ \sigma^{t2}) \tag{13}$$

The solution algorithm was as follows: the geometry of the lamina was defined and the strain and curvature distribution were imposed incrementally. At each increment of the strain, the stiffness was calculated and used to calculate the stress. The stress was checked against the failure criterion. If the failure criteria were met, then the stress level and the stiffness of that layer were adjusted according to the constitutive response. Subsequent loadings of a cracked layer resulted in a change in the magnitude of the damage parameter. This indicates that at any stress level, the degradation of elastic properties was primarily related to the magnitude of crack density and overall strain response. Using the updated damage, the quasi-elastic stiffness parameters A, B, and D were obtained and used to calculate the load and moment for that increment.

EXPERIMENTAL PROGRAM

A test program was conducted to measure the mechanical response of glass/epoxy and Carbon/Epoxy composites under uniaxial tension and fatigue tests [Young, and Mobasher, 1996]. A description of the mode of failure and the strength of each sample were presented in detailed report. Tests were conducted at the Mechanical Testing of Materials Laboratory at the College of Engineering and Applied Sciences. Uniaxial tension tests in the fiber direction and transverse to the fiber direction were conducted. A Servohydraulic closed loop testing system with a total capacity 55 kips was used. The load vs. deformation of the specimens were collected. Two types of displacements were measured. The strain as measured from the extensometer mounted across a 1.5 in gage length, and the total elongation of the specimen as measured by the cross-head displacement. Data was collected using a digital data acquisition system on a continuous basis throughout the test. All of the test data were analyzed by means of digital data processing software.

DISCUSSION OF RESULTS

The properties of matrix and fibers are shown in Table 1. were used in the rule of mixtures to calculate the longitudinal and transverse modulus E_1, E_2 and Poisons ratio ν_{12} was achieved using the Halpin-Tsai predictions shown in Equation 2.

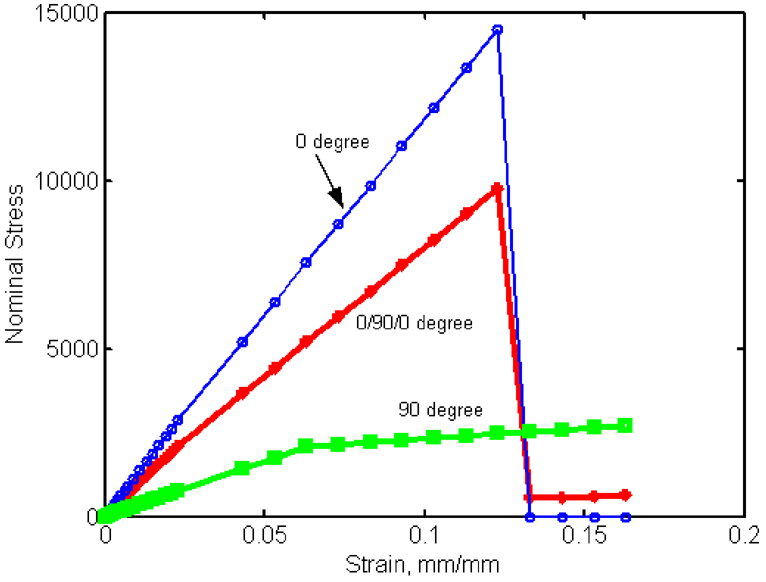

Figure 2. Nominal Stress Strain response of unidirectional, 0/90/0, and 90 degree glass–epoxy composite laminates

	V_f (%)	Ultimate Strength (Ksi)	Elastic Modulus (Ksi)	Poison's Ratio
Glass Fibers epoxy, 0, 90 degrees	45	$\sigma_{fu} = 240$ $\sigma_{mu} = 10$	E_f=10500 E_m= 600	$\upsilon_f = 0.2$ υ_m=.18
Woven Carbon Textile	50	σ_{t1}=5-10	E_f= 100000	υ_m=.25

Table 1. Material property of Glass and Carbon fiber composites studied.[1]

Figure 2 shows the response of unidirectional, 0/90/0, and 90 lamina with glass epoxy to a uniformly applied strain. Note that as the ultimate strength of the matrix phase is reached, there is a shift in the slope of the stress strain response. As the damage accumulation increases, it results in a reduction in stiffness for the overall composite. The load carrying capacity extends well beyond the matrix-cracking phase and as damage accumulates the stiffness gradually decays. Model predictions for the effect of lamina orientation are also shown in the figure. As a 0 degree lamina is replaced by 90 degree layers, it is observed that both the first crack strength and also the post cracking stiffness drop markedly.

Figure 3 shows the comparison of theory and experimental data for carbon epoxy system subjected to uniaxial stress. The parameters of the model were changed to obtain a reasonable fit to the experimental results. Note the using the present system, one can conduct a back calculation procedure to use the experimental stiffness and strength measures in order to compute the equivalent orthotropic properties of the composite.

Figure 4 represent the comparison of the present approach with the experimental data of 0 and 90 degree stacked lamina to a uniformly applied tensile strain level. A constant strain level is imposed in stages across the depth of the cross section. Note that the stress in the longitudinal layers increases to a maximum level determined by the fiber fracture strength. The maximum load is attained when the stress in longitudinal lamina reaches a stress equal to the effective strength of the fiber phase, or $V_f\ \sigma_{fu}$. In the transverse direction as shown in Figure 4, the stiffness and strength are both significantly lower than the 0 degree layers. The loading in the transverse direction is limited to the ultimate tensile strength σ_{2t}. As the stress in both orientations exceeds the strength of the matrix, the damage parameter is increased. The stiffness degradation due to the damage parameter results in non-linear response which is also shown in the load vs. deformation level.

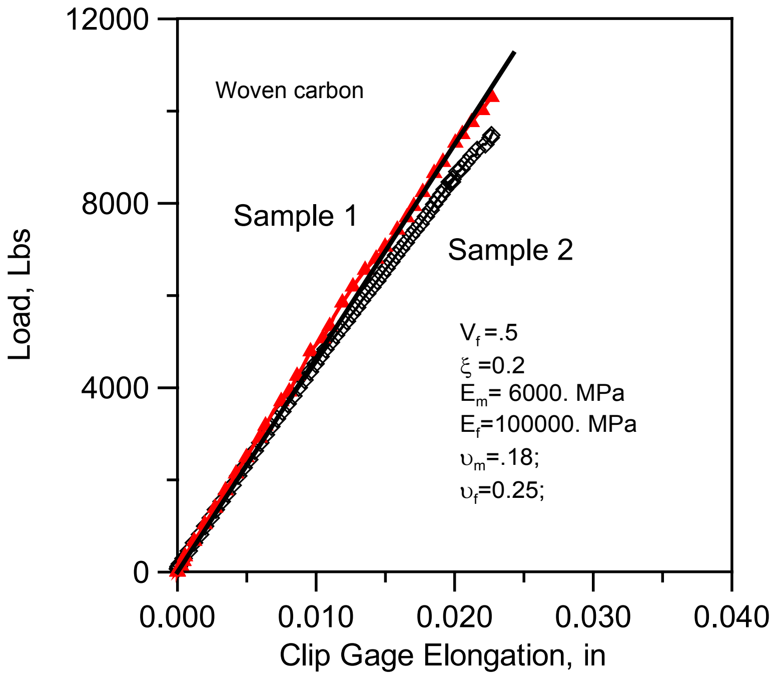

Figure 4. Nominal Stress Strain response of unidirectional, 0/90/0, and 90 degree glass–epoxy composite laminates

CONCLUSION

A theoretical model is presented to predict the response of composite laminates subjected to axial loads. The model utilizes composite laminate theory subjected to material degradation by means of a scalar damage parameter. Theoretical results are compared to experimentally obtained data and indicate a good agreement for several lamina configurations, and composites.

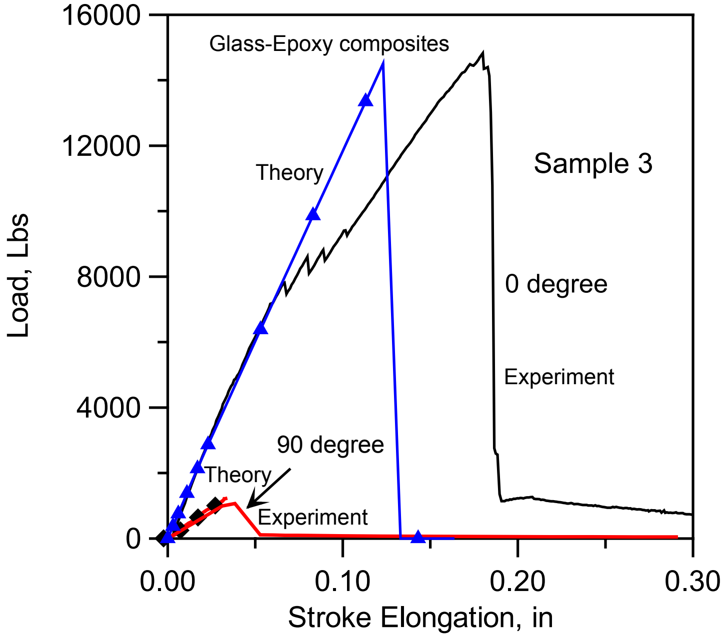

Figure 3. Comparison of theortical and experimental Stress Strain response of unidirectional, and 90 degree glass–epoxy composite laminates

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