



HYSTERESIS OF CAPILLARY COHESION IN UNSATURATED SOILS

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ABSTRACT

A theoretical analysis is conducted for modeling the constitutive relationships among water content, matric suction, and capillary cohesion in unsaturated granular soils (*e.g.*, sands or silts). A rigorous series of equations is developed to describe the interparticle forces due to negative pore-water pressure for idealized spherical particles in simple-cubic and tetrahedral packing geometries. By considering changing meniscus geometry, matric suction and effective stress are evaluated as functions of water content for soil particles under the pendular (*i.e.*, disconnected) pore-water regime. The contact angle at the interface between the pore-water menisci and the solid soil particles is considered as an arbitrary variable so that its effects on the hysteretic behavior of matric suction, effective stress, and capillary cohesion may be evaluated. The analysis provides a theoretical basis for describing several well-known phenomena in unsaturated soil behavior. Varying the contact angle from 0° to 40° to simulate drying and wetting processes respectively is shown to have an appreciable impact on hysteresis in the constitutive behavior of the modeled soils. The observations from the modeling effort are of practical importance in developing an improved understanding of the behavior of real unsaturated soils undergoing natural wetting and drying processes such as infiltration or drainage. This paper presents partial results of the described research.

Keywords: hysteresis, capillary cohesion, unsaturated soils, suction, contact angle

INTRODUCTION

Capillary cohesion is an unsaturated soil mechanics concept referring to the net interparticle force generated within a matrix of granular particles (*e.g.*, silt or sand) due to the combined effects of negative pore water pressure and surface tension. The macroscopic consequence of capillary cohesion is a force that tends to pull the soil grains towards one another, similar in effect and sign convention to an overburden stress or surcharge loading. The important role of capillary cohesion in the shear strength, deformation, and permeability behavior of unsaturated soils has long been recognized. In recent years, analyzing and accounting for capillarity in the context of traditional geotechnical engineering problems including slope stability, bearing capacity, and lateral earth pressure have become the subjects of much research in the rapidly growing field of

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unsaturated soil mechanics (*e.g.*, Fredlund and Rahardjo, 1993).

Hysteresis is a well-known phenomenon in many aspects of unsaturated soil behavior. Perhaps the most outstanding example is the commonly observed hysteresis between drying and wetting loops in the constitutive relationship between soil suction and water content (*i.e.*, the soil-water characteristic curve). In general, unsaturated soils undergoing drying processes such as evaporation or drainage tend to retain a greater amount of water than at the same magnitude of suction during wetting processes such as infiltration or capillary rise. It follows then, that if capillary cohesion is directly dependent on the magnitude of matric suction (*i.e.*, negative pore water pressure), there is likely to be some amount of wetting-drying hysteresis in the relationship between capillary cohesion and water content. In practical engineering situations, where wetting and drying processes occur quite often, there is a strong motivation to understand this possible hysteretic behavior and its impact on the behavior of manmade or natural unsaturated soil systems.

The objective of this research is to examine several aspects of hysteretic behavior in the interparticle capillary cohesion forces between unsaturated granular soil particles. A theoretical approach is adopted by considering resultant interparticle forces arising from negative or positive pore water pressure and surface tension in the curved water menisci between unsaturated spherical particles. Simple-cubic and tetrahedral grain packing geometries are selected as end members representative of the wide range of void ratios in typical real soils. Hysteresis in the relationships among water content, soil suction, and capillary cohesion are evaluated by developing a series of analytical equations with the flexibility to account for differences in the contact angle at the solid-water interface during wetting and drying.

BACKGROUND

Early attempts at understanding capillarity and its role in the constitutive stress-strain behavior of unsaturated soils recognized that when soil is saturated, the pore water pressure is positive and the net effect of the water pressure is to push the soil grains apart. At the opposite extreme, however, when the soil is nearly dry, it was recognized that the remaining water in the voids may sustain very high negative pore pressures, thus creating tensile forces acting to pull the soil grains together. The resultant interparticle stress in the range between these extremes was described in the form of Terzaghi's classic effective stress equation, slightly modified to account for the negative pore water pressures (*e.g.*, Bishop, 1959, 1961; Donald, 1961; Blight, 1961). Bishop (1959), for example, proposed a single-valued effective stress equation for unsaturated soils in the form:

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) \quad (1)$$

where σ' is the effective interparticle stress, σ is the total stress, u_a is the pore air pressure, u_w is the pore water pressure, the quantity $(u_a - u_w)$ is matric suction, and χ is a constitutive material property of the soil that depends on degree of saturation. The term $(\sigma - u_a)$ in equation (1) represents the traditional component of effective stress applicable to saturated soils. The product $\chi(u_a - u_w)$, on the other hand, represents the interparticle effective stress due to capillary cohesion.

THEORETICAL DEVELOPMENT

Idealized Grain Geometries

The analysis of water content, pore pressure, and capillary cohesion in unsaturated sands is

greatly simplified by assuming that the soil grains are perfectly spherical and arranged according to some idealized packing geometry. In the following analysis, simple cubic (SC) and tetrahedral (TH) packing geometries are considered to represent two end members in granular soil fabric. It is presumed that the range in the material properties and behavior of real sands falls somewhere between these two extreme scenarios.

Capillary Force Between Two Particles

In two dimensions, any two contacting particles can be considered as an elementary volume. Referring to Figure 1a, the common radius of the contacting particles is R , which for sand or silt grains may range anywhere between approximately 1 mm and 1×10^{-3} mm. Two additional radii, r_1 and r_2 , are used to define the geometry of the water meniscus, or “lens,” that forms between the particles under unsaturated conditions. The radius r_1 , rotated about its axis perpendicular to the cross section shown, defines the concave curvature of the water meniscus. The radius r_2 , rotated about its axis parallel to the cross section, defines the convex curvature of the meniscus in the third dimension. The angle θ , designated herein as the “water content index angle,” connects the center of either soil particle to the center of the circle defined by r_1 . Increasing or decreasing the index angle allows us to describe changes in the size of the water lens with increasing or decreasing saturation.

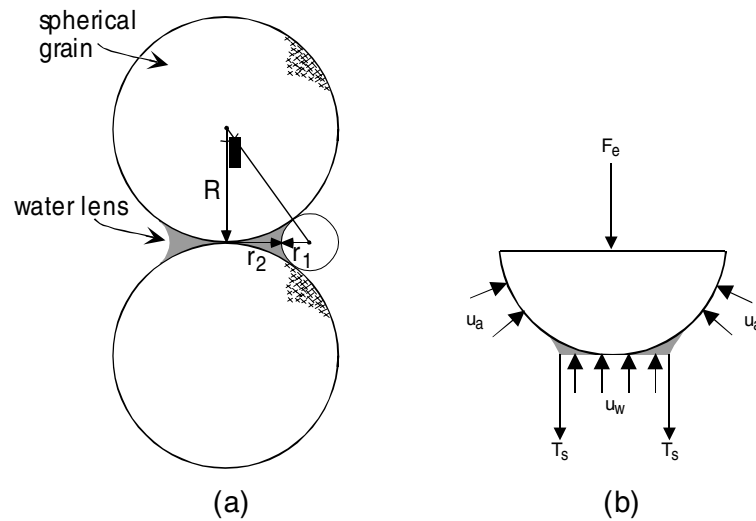


FIG. 1. Free body diagrams for geometry of interparticle water lens

The free body diagram shown as Figure 1b is cut through the center of the water lens at the particle contact point to illustrate the interparticle forces and stresses acting on the unsaturated grains. The pore air pressure u_a will be upward, resulting in a force on the air-solid interface equal to:

$$F_a = u_a (\pi R^2 - \pi r_2^2) \quad (2)$$

At the mid-plane between the particles, the force due to surface tension along the air-water interface is vertical. The total force due to surface tension is equal to the product of the meniscus circumference and the surface tension of the water T_s , which can be written as:

$$F_t = -T_s 2\pi r_2 \quad (3)$$

The projection of total force due to water pressure u_w acting on the water-solid interface in the vertical direction is:

$$F_w = u_w \pi r_2^2 \quad (4)$$

The resultant capillary force between the particles is the sum of all the above three forces:

$$F_e = u_a \pi R^2 - (u_a - u_w) \pi r_2^2 - T_s 2\pi r_2 \quad (5)$$

where the term $(u_a - u_w)$ is matric suction. For u_a set to a reference value of zero and for negative pore-water pressures, F_e is a negative number, indicating that the macroscopic effect of capillarity in unsaturated soils is to compress the granular soil matrix, creating what has been often referred to as an “effective cohesion” in otherwise cohesionless soils.

WATER CONTENT BETWEEN TWO CONTACTING PARTICLES

The water lens radii r_1 and r_2 can be written in terms of the water content index angle θ , the common particle radius R , and the contact angle α as follows:

$$r_1 = R \frac{1 - \cos \theta}{\cos(\theta + \alpha)} \quad (6)$$

$$r_2 = R \tan \theta - r_1 \left(1 - \frac{\sin \alpha}{\cos \theta} \right) \quad (7)$$

The total volume of the water lens in simple cubic packing order is:

$$w_{sc} = \frac{9}{2G_s} \sin^2 \theta - \frac{9}{2G_s} \sin^2 \theta \cos \theta - \frac{3}{2G_s} (1 - \cos \theta)^2 (2 + \cos \theta) - \frac{9V_r}{4G_s \pi R^3} \quad (8)$$

where G_s is the specific gravity of the soil solids and,

$$V_r = 2\pi \left[r_2 + r_1 - \frac{2}{3} \frac{r_1 \cos^3(\theta + \alpha)}{\frac{\pi}{2} - (\theta + \alpha) - \sin(\theta + \alpha) \cos(\theta + \alpha)} \right] \frac{1}{2} r_1^2 [\pi - 2(\theta + \alpha) - \sin 2(\theta + \alpha)]$$

In tetrahedral (TH) packing, the water content for a given index angle θ is simply twice that of SC packing for the same index angle:

$$w_{TH} = 2w_{SC} \quad (9)$$

PORE PRESSURE REGIMES

Matric suction within the water lens is described by the well-known Laplace equation:

$$(u_a - u_w) = T_s \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (10)$$

For u_a equal to zero, equation (10) dictates that a decrease in the radius r_1 results in more negative values of pore water pressure, a reflection of its relationship to the concave curvature of the water lens. A decrease in r_2 , on the other hand, causes the pore water pressure to be less negative, a reflection of its relationship to the convex curvature of the water lens.

If $r_1 < r_2$, then pore water pressure less than the air pressure develops in the lens (*i.e.*, a negative value for a reference air pressure equal to zero). If $r_1 > r_2$, a positive water pressure develops. Finally, if $r_1 = r_2$, which must occur at some value of water content, the pressure in the water lens, and thus the matric suction, are equal to the zero. Considering equations (6) and (7), this occurs for an arbitrary contact angle when:

$$(1 - \cos \theta)(2 \cos \theta - \sin \alpha) = \sin \theta \cos(\theta + \alpha) \quad (11)$$

This equation, as plotted on Figure 2, identifies the boundary between the negative pore water pressure regime and the positive pore water pressure regime.

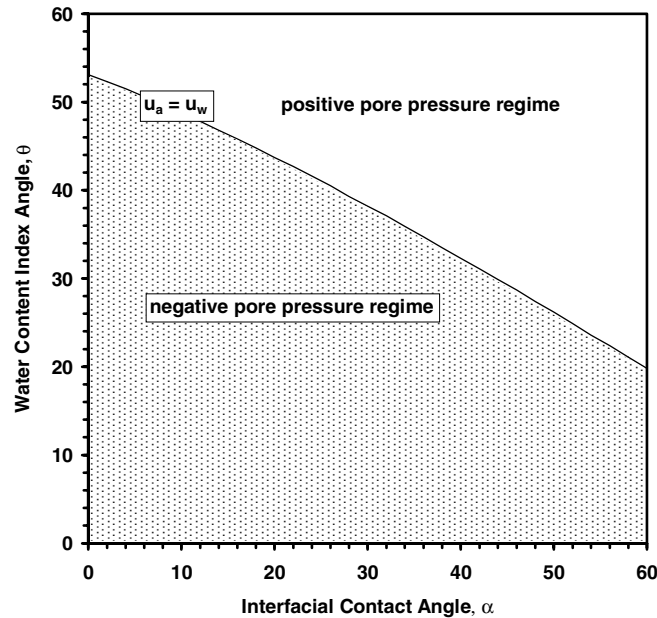


FIG. 2. Regimes of positive and negative pore water pressure

HYSTERESIS IN THE SOIL-WATER CHARACTERISTIC CURVE

The preceding analysis can be used to examine the relationship between matric suction and water content, *i.e.*, the soil-water characteristic curve (SWCC), for spherical particles in simple cubic and tetrahedral packing order. For θ ranging from 0° to 45° , equations (8) and (9) can be used to calculate water content in SC and TH packing respectively. Given a fixed particle diameter R and contact angle α , matric suction values corresponding to the calculated water content values can be determined from equations (6), (7) and (10).

Figure 3a shows matric suction characteristic curves calculated in this manner for three values of R , ranging from 0.1 mm to 1.0 mm, and a zero contact angle. As is the case for real soils, the larger the particle size, the less the magnitude of suction for the same value of water content.

Figure 3b shows characteristic curves for soils modeled with non-zero contact angles in SC packing order. The larger contact angles (which may simulate a wetting process) result in less water retained by the soils than at the same value of suction for lower contact angles (simulating a drying process). This hysteretic behavior is identical to that observed in typical characteristic curves for real soils.

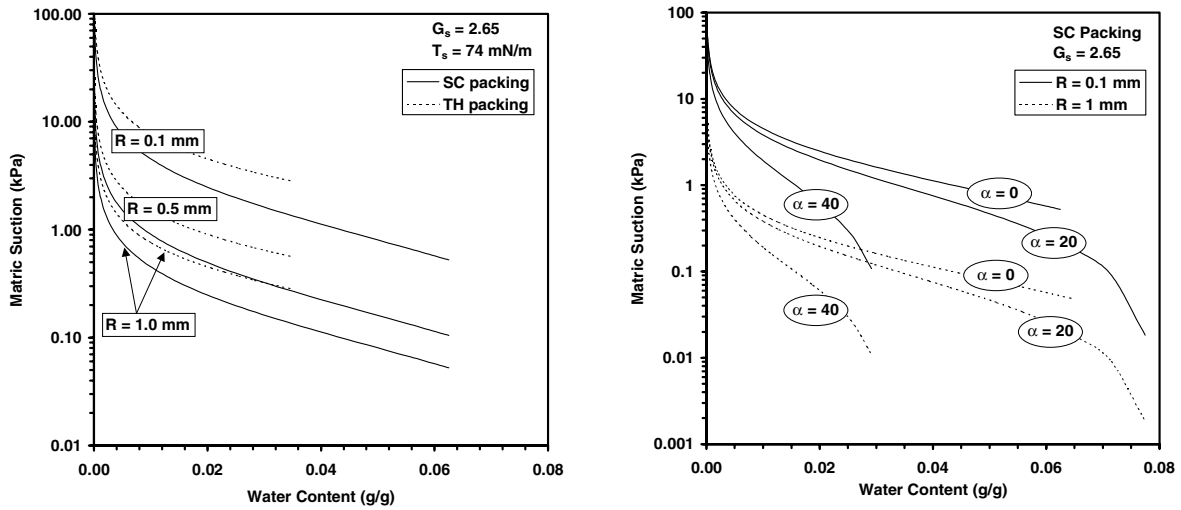


FIG. 3. Soil-water characteristics for a) zero and b) non-zero contact angles

HYSTERESIS IN CAPILLARY COHESION

The net capillary stress as a function particle size and the water lens radii r_1 and r_2 can be written:

$$\sigma_w = -\frac{r_2^2}{R^2} \left(\frac{r_2 + r_1}{r_2 - r_1} \right) \left(\frac{r_2 - r_1}{r_2 r_1} \right) T_s = -\frac{r_2^2}{R^2} \left(\frac{r_2 + r_1}{r_1} \right) T_s \quad (12)$$

The total capillary force between the two particles is:

$$F_{cap} = \sigma_w \pi R^2 = -\frac{r_2 + r_1}{r_1} \pi r_2 T_s \quad (13)$$

Figure 4 shows the relationship between water content and capillary cohesion (equation 12) for $R = 1$ mm (Figure 4a) and $R = 0.1$ mm (Figure 4b). Capillary cohesion increases by an order in magnitude with a decrease in particle size of the same order of magnitude. The greater tendency for large capillary forces to develop between relatively fine-grained soils may explain the much greater tendency of fine-grained soils to shrink during drying.

Contact angle is varied from 0° to 40° so that the effects of contact angle hysteresis may be evaluated. For $\alpha = 0^\circ$, the capillary cohesion function agrees quite well with a previous analysis by Cho and Santamarina (2001). It is immediately apparent that increasing the contact angle has a significant effect on the magnitude of capillary cohesion. The larger the angle, the less the cohesion. This observation may have important practical implications. For example, this would indicate that soils undergoing a wetting process (consider an unsaturated slope during a precipitation event) have less of a contribution to effective stress from capillarity, and thus shear strength, than during a similar drying process.

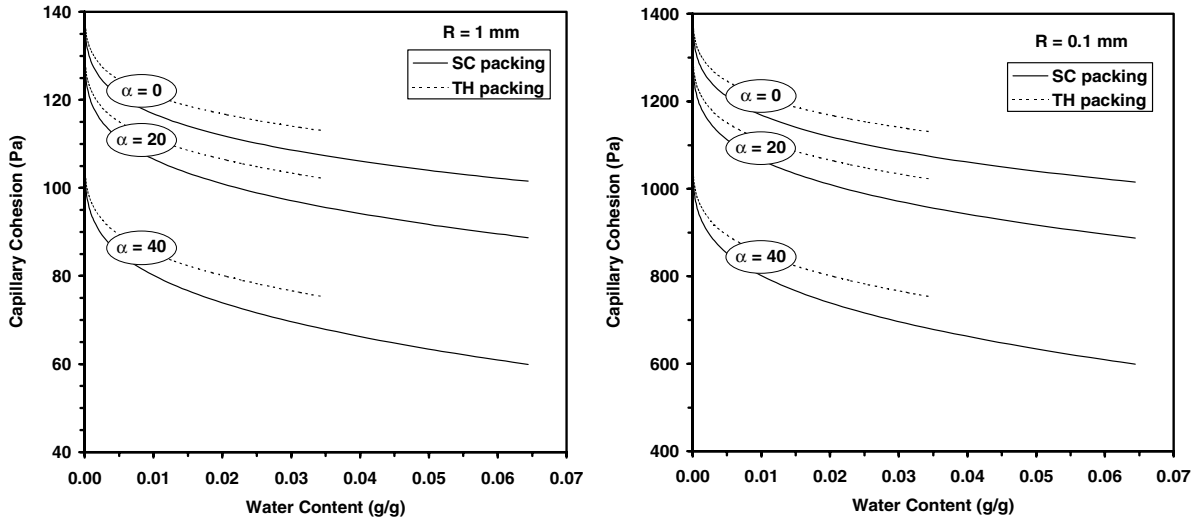


FIG. 4. Hysteresis in capillary cohesion for a) $R = 1$ mm and b) $R = 0.1$ mm

SUMMARY AND CONCLUSIONS

A theoretical investigation was performed to evaluate hysteretic behavior in the constitutive

relationships among water content, matric suction, and capillary cohesion in unsaturated granular soils (e.g., sands or silts). Hysteresis in the contact angle at the pore water and soil particle interface between wetting and drying was considered as the primary mechanism responsible for hysteretic behavior in unsaturated soils. A series of rigorous analytical equations was developed for describing the total volume and pressure of menisci between contacting spherical soil grains by treating the contact angle as an arbitrary material variable. Corresponding water contents were calculated for simple cubic and tetrahedral grain packing geometries.

Matric suction characteristic curves modeled using the analytical solutions displayed behavior similar to real unsaturated soils. Varying the contact angle caused significant hysteresis in the characteristic curves. Relatively large contact angles, selected to model wetting processes, resulted in significantly less values of water content for the same value of suction as those modeled for relatively small contact angles. Varying the contact angle showed that relatively large contact angles result in significantly lower capillary forces, indicating that the contribution of capillarity to shear strength for soils undergoing wetting processes such as infiltration may be lower than that for soils experiencing drying processes such as drainage.

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REFERENCES

- Bishop, A.W., 1959, "The principle of effective stress," *Teknisk Ukeblad I Samarbeide Med Teknisk*, Oslo, Norway, 106(39), pp. 859-863.
- Bishop, A.W., 1961, "The measurement of pore pressure in the triaxial test," *Conf. British Nat. Soc. Of Int. Soil Mech. and Found. Engrg.*, Butterworth's, London., pp. 38-46.
- Bishop, A.W., Alpan, I., Blight, G.E., and Donald, I.B., 1960, "Factors controlling the shear strength of partially saturated cohesive soils," *ASCE Conf. Shear Strength of Cohesive Soils*, Boulder, CO, pp. 503-532.
- Blight, G.E., 1961, "Strength and consolidation characteristics of compacted soils," Ph.D. Dissertation, University of London, London, England.
- Blight, G.E., 1967, "Effective stress evaluation for unsaturated soils," *ASCE Journal of the Soil Mechanics and Foundations Division*, Vol. 92, SM2, pp. 125-148.
- Cho, G.C, and Santamarina, J.C., 2001, "Unsaturated particulate materials: particle-level studies," *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol. 127, No. 1, pp. 84-96.
- Dallavalle, J.M., 1943, *Micromeritics*, Pitman, London.
- Donald, I.B., 1961, "The mechanical properties of saturated and partly saturated soils with special reference to negative pore water pressure," Ph.D. Dissertation, University of London, London, England.
- Fredlund, D.G., and Rahardjo, H., 1993, *Soil Mechanics for Unsaturated Soils*, Wiley Inter-Science.
- Sparks, A.D.W., 1963, "Theoretical considerations in stress equations for partly saturated soils," *Proc. 3rd Regional Conference for Africa on Soil Mechanics*, Salisbury, Rhodesia, Vol. 1, pp. 215-218.