IASSAR SC2: Stochastic Stability

Christian Bucher
Institute of Structural Mechanics
Bauhaus-Universität Weimar, Germany

July 22, 1999

Dynamical systems are typically described by a system of differential equations in the following form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{p}, t, \omega) + \mathbf{g}(t, \omega)$$

In this equation, $\mathbf{x}$ denotes a state vector, t is time, ω denotes randomness, $\mathbf{f}$ is a nonlinear function (including the effect of parametric excitation) depending on $\mathbf{x}$, on a set of parameters $\mathbf{p}$, on t, and on ω, and $\mathbf{g}$ is external load.

Stability in the Lyapunov sense addresses the effect of small disturbances on the trajectories of a dynamical system. Asymptotic stability implies that the effect of such disturbances will eventually vanish. In deterministic stability analysis the question of local stability is answered by determining the top Lyapunov exponent λ of the tangential system. This implies linearization of the original dynamical system about the reference solution (whose stability is to be investigated).

Stochasticity in stability problems of structural dynamics arises from two sources. First, the system properties $\mathbf{p}$ may be randomly fluctuating in space, or be simple random variables. Second, the load $\mathbf{g}(t)$ acting on the system may be a random process in time. In both case, the question of stability can be only answered in terms of probabilistic measures such as expected values or distribution functions.

The concept of the Lyapunov exponent is easily carried over to the first case as mentioned above. The analysis is performed using the concept on conditional probability, i.e. the top Lyapunov exponent λ of the system is conditional on the actual values of the time-invariant system parameters. Taking into account the distribution of $\mathbf{p}$ in principle arbitrary statistics of λ can be derived. From a computational point of view, however, this might become infeasible. This is particularly true, if the parameters $\mathbf{p}$ constitute a random field with partial correlation within the structure. In this case, it is essential to perform a suitable discretization of the random field and reduction of the number of variables. The methodology involves a spectral representation (e.g. Karhunen-Loève expansion) and truncation. Examples can be found in the effect of random geometrical imperfections of structures on stability.

The second case requires somewhat more attention. Due to the random time-variance of the tangential system the notion of asysmptotic stability needs to be interpreted in a statistical sense. This leads to stability definitions such as

- Stability in probability
- Stability in mean square
- Almost sure Stability

In addition, the notion of structural stability is quite helpful. In the present context, this means the persistence of a particular shape of the probability density. This implies that loss of stability (or bifurcation) leads to a sudden change of the probabilistic structure of the response. Available methods to deal with this kind of problems are almost exclusively based on Markov vector theory. This means that the original system of equations is replaced by an equivalent system of Itô equations. This systems is then analyzed with respect to stability.

Provided solutions to both cases are available the combined problem can be dealt with quite easily, since usually it can be assumed that system parameters and load are statistically independent.