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Seismic analysis of sliding wedge: extended Francais–Culmann's analysis

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Abstract

Pseudo-static analysis is commonly used to design earth structures. Most pseudo-static methods of analysis require a computer program. This paper presents a simple closed form solution of seismic stability analysis by extending Francais–Culmann's analysis. The analysis is valid for a slope with the most critical planar mechanism and under the influence of horizontal and vertical earthquake accelerations. The resulting permanent displacement is discussed and demonstrated using several real earthquake records. The simplicity of extended Francais–Culmann's solution allows it to be used for preliminary design and classroom instruction. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Slopes; Seismic stability; Yield acceleration; Permanent displacement; Vertical acceleration

1. Introduction

Seismic designs of geotechnical earth structures, such as slopes, retaining walls, embankments and dams, are conducted routinely using a pseudo-static approach. The Mononobe–Okabe (M–O) approach [1,2], for retaining wall design, is the most well-known pseudo-static procedures. It is considered an earth pressure approach where the solution is obtained by extending Coulomb's analysis. Another commonly used pseudo-static approach is for the slope stability analysis. In the seismic slope stability analysis, the use of a circular and log-spiral failure surface has been proposed [3–7]. As a result of its complexity, these methods of slope stability analysis are typically conducted using a computer program. Thus, the effects of different basic parameters on the design outcome are not readily known at a glance.

Pseudo-static stability analysis that uses a mechanism at a prescribed failure plane has been addressed by several investigators [8–10]. These studies all assume the inertia force due to an earthquake horizontal acceleration for a failure soil mass along a prescribed plane. Recent earthquakes recorded significantly large vertical accelerations: 1994 Hokkaido Toho-Oki Earthquake [11] ($M = 7.9$), 1989 Loma Prieta Earthquake [12] ($M = 7.1$), 1994 Northridge Earthquake [13] ($M = 6.7$), and 1995 Hanshin Earthquake [14] ($M = 7.2$). For certain measuring stations located close to the faults, the vertical acceleration was as large as that of horizontal acceleration. However, the effects of the vertical acceleration on the seismic performance of earth structures are not readily known. Thus, an analysis incorporating vertical acceleration and that which provides a quick solution is preferred.

In this paper, Francais–Culmann's method [15,16] of wedge stability analysis is extended to seismic conditions. With the closed form solution expressed using simple equations, the effects of design parameters are revealed with ease. Thus, combined with permanent displacement analysis, the methodology is of significance for classroom instruction. It is well known that Francais–Culmann's method is applicable to steep slopes and thus this solution would also be beneficial for preliminary slope design.

2. Stability analysis

This paper considers a wedge that slides along the most critical planar surface. The procedure for static stability analysis is commonly known as Culmann's analysis. Note, however, Golder [17] pointed out that Francais [18] was the first one to have presented such an analysis.

Fig. 1(a) shows the mechanism of stability analysis. A homogeneous slope of height H and inclination i is considered. The backfill of this slope is inclined at an angle β. The unit weight of soil is expressed as γ. The soil wedge is assumed to be rigid and slide along a planar surface. The seismic inertia force, whose magnitude and direction vary throughout the duration of excitation, is regarded as pseudo-static by representing it using horizontal and vertical seismic coefficients: k_h and k_v. Note that positive k_v is considered to act downwards in this study.

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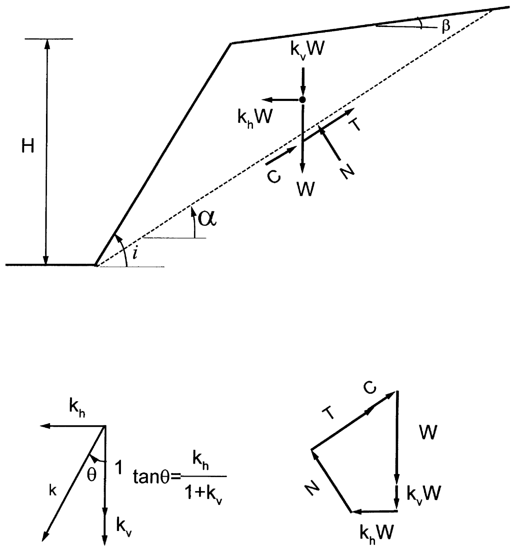

Fig. 1. Seismic stability analysis with planar failure surface.

The analysis presented in this paper, however, seeks for the most critical failure plane. It is assumed that soil obeys Coulomb failure criterion, expressed by two strength parameters: cohesion c and angle of internal friction ϕ. In design, cohesion and frictional components of the strength are reduced by a designated factor of safety, $F_{s.}$ They are usually expressed as dimensionless parameters, N_m and ψ_m [19]

$$N_m = \frac{c}{F_s}\frac{1}{\gamma H}, \tag{1}$$

$$\psi_m = \frac{\tan\phi}{F_s}. \tag{2}$$

In this paper, considering a factor of safety of unity, N_m and ψ_m are expressed as N and ψ, respectively. The effect of pore water pressure is not included, thus the formulations are valid for free-draining soils or for a total stress analysis.

From Fig. 1(a), the force equilibrium equations along and normal to the failure plane are written as

$$N = \{(1 + k_v)\cos\alpha - k_h\sin\alpha\}W, \tag{3}$$

$$T = \{(1 + k_v)\sin\alpha + k_h\cos\alpha\}W, \tag{4}$$

where N and T are the normal and shear forces acting along the failure plane, respectively. The dead weight of failure soil mass, W, and the length of the planar failure plane, l, are given as

$$W = \frac{\gamma H^2}{2}\frac{\sin(i-\alpha)\cos\beta}{\sin(\alpha-\beta)\sin i}, \tag{5}$$

$$l = \frac{\sin(i-\beta)}{\sin(\alpha-\beta)\sin i}H. \tag{6}$$

The average normal and shear stresses acting on the failure plane are determined by dividing Eqs. (3) and (4) by the length of failure plane (Eq. (6)), and then substituting them into Coulomb failure criterion,$\tau = c + \sigma\tan\phi$. The cohesion required to restore equilibrium is thus obtained

$$c = \frac{\gamma H(1+k_v)}{2}\frac{\cos\beta\sin(\phi-\alpha-\theta)\sin(i-\alpha)}{\cos\theta\cos\phi\sin(i-\beta)}. \tag{7}$$

The critical inclination of failure plane, α_{cr}, is determined when a maximum cohesion is required to restore equilibrium, i.e. at $dc/d\alpha = 0$. α_{cr} is obtained as

$$\alpha_{cr} = \frac{i+\phi-\theta}{2}, \tag{8}$$

where

$$\tan\theta = \frac{k_h}{1+k_v} \tag{9}$$

θ shows the inclination of resultant acceleration and is represented geometrically in Fig. 1(b). It can be seen from Eq. (8) that under seismic conditions, the critical angle is reduced for a slope, and thus a larger soil mass slides along the failure plane compared to the static case.

The cohesion required to restore equilibrium at this critical failure plane is obtained by substituting Eq. (8) into Eq. (7). It is then normalized to give a stability number, N

$$N = \frac{c}{\gamma H} = \frac{(1+k_v)}{4}\frac{\cos\beta(1-\cos(\phi-i-\theta))}{\cos\theta\cos\phi\sin(i-\beta)}. \tag{10}$$

In the absence of seismic accelerations and for a horizontal backfill, Eq. (10) degenerates to that of Francais–Culmann solution [15,16]

$$N = \frac{1-\cos(i-\phi)}{4\sin i\cos\phi}. \tag{11}$$

3. Permanent displacement

As presented earlier, the factor of safety is used to design or assess slope stability in a pseudo-static approach. While this is a convenient design procedure, physical interpretation of this factor of safety is not apparent under seismic loading conditions since the time response of a slope is not considered. In the occasions of design for large earthquake, as required for Level 2 seismic design proposed in Japan [20], where seismic acceleration will exceed $0.7 \sim 0.8g$, it becomes uneconomical to design slopes based on solely a factor of safety.

On the contrary, sliding block theory proposed by Whitman [21] and Newmark [22] is considered a more appropriate tool to assess slope performance. It allows performance to be determined based on permanent displacement. In the sliding block theory, the yield acceleration of the slope is defined as the acceleration when the soil mass starts to slide. The relative acceleration of the soil mass is

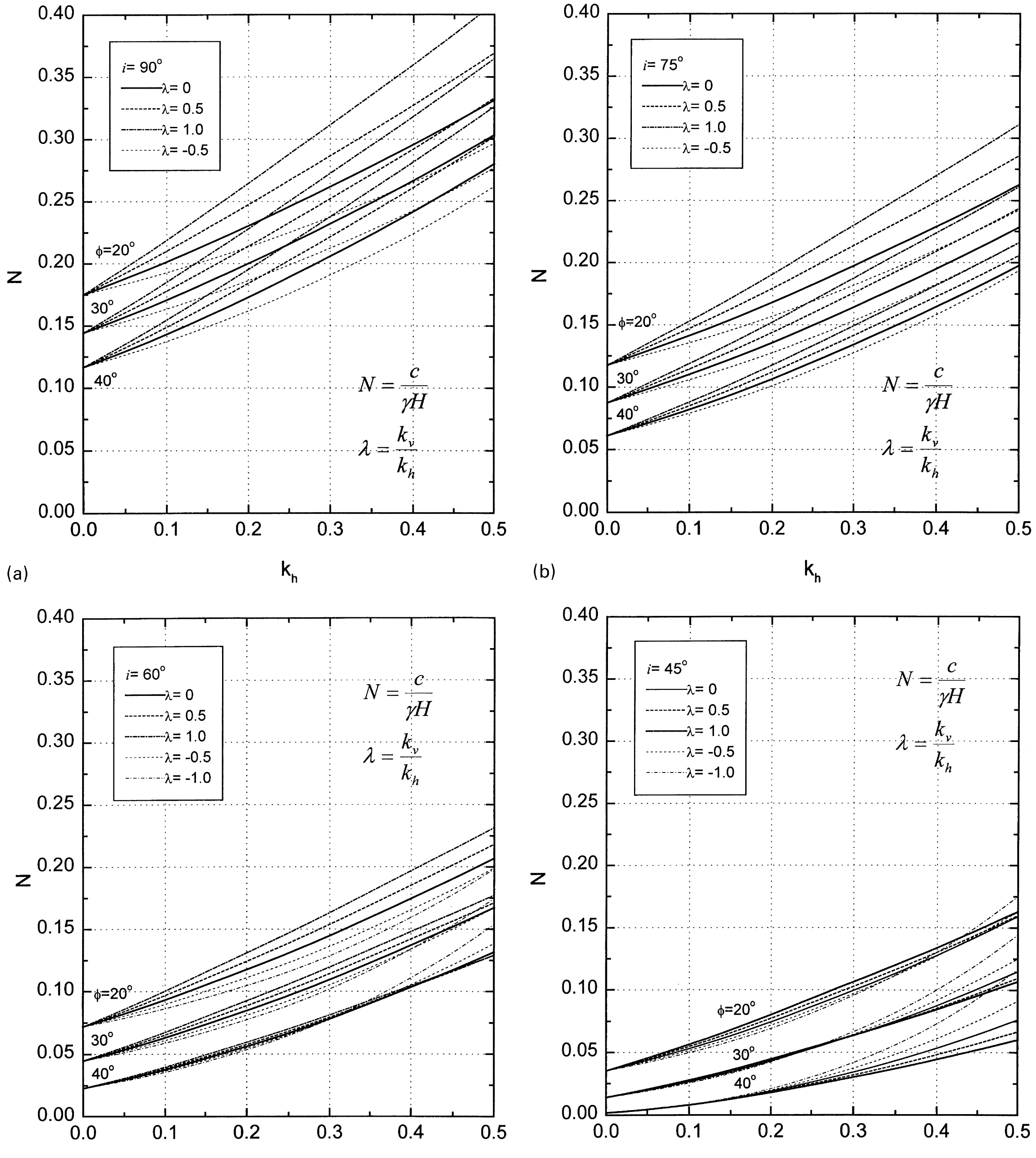

Fig. 2. Relationships between soil strength and seismic coefficients: (a) $i = 45°$; (b) $i = 60°$; (c) $i = 75°$; and (d) $i = 90°$.

then double-integrated to give permanent displacement. The reverse yield acceleration is sufficiently large such that its effects may be neglected in calculating for permanent displacement.

Permanent displacement of slopes has been examined to a certain extent [7,23–27]. Goodman and Seed [23] considered a planar failure surface of cohesionless soil. Sarma [24] used a circular mechanism while Sawada et al. [26] and Ling and Leshchinsky [27] used a log-spiral mechanism. Franklin and Chang [28], Haynes and Franklin [29], Makdisi and Seed [30] compiled design charts from a large number of earthquake records for estimating possible permanent displacement. While different failure mechanisms were examined, the effect of vertical acceleration has not been addressed adequately in the context of permanent displacement.

In the planar mechanism as discussed earlier, the yield seismic coefficient is determined from the equation of

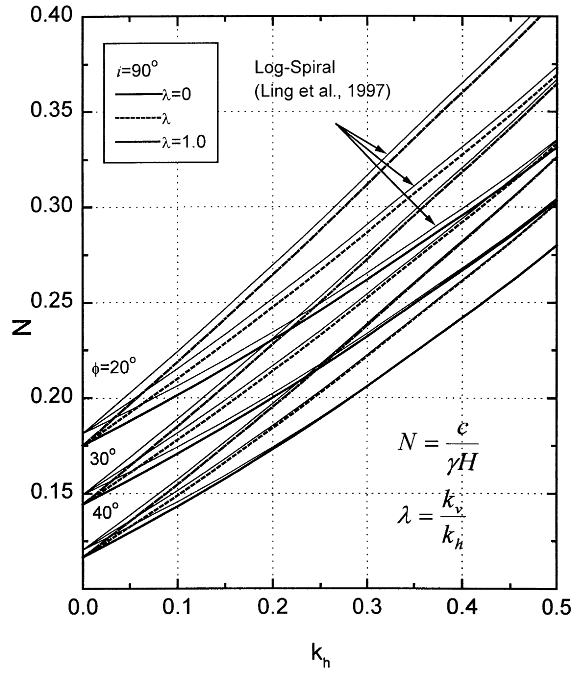

Fig. 3. Comparison of N between planar and log-spiral mechanism for vertical slopes.

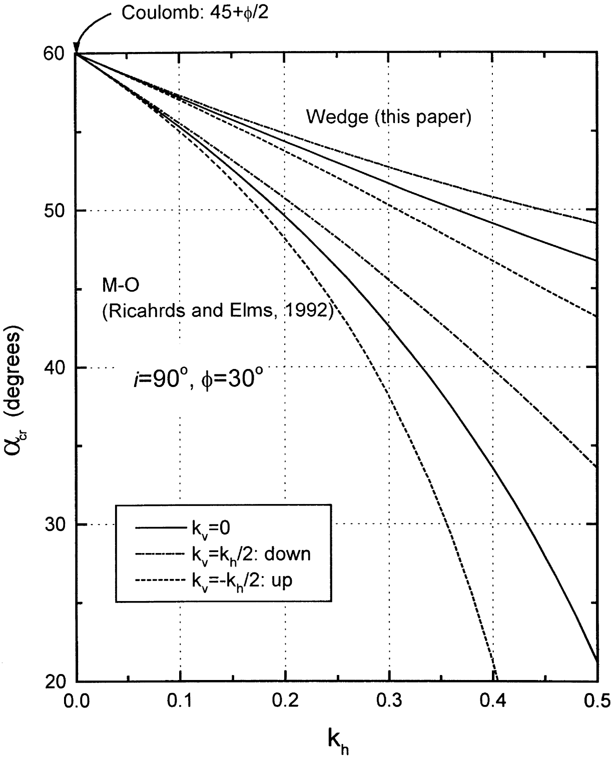

Fig. 4. Comparison of critical failure plane for wedge stability analysis and Mononobe–Okabe analysis.

equilibrium. At the instant of sliding

$$k_{\mathrm{hy}} = (1 + k_{\mathrm{v}})\tan(\phi - \alpha) + \frac{\sin i \cos \phi}{\sin(i - \alpha)\cos(\phi - \alpha)} \frac{2c}{\gamma H}. \tag{12}$$

The acceleration of soil mass along the failure plane, relative to the stable base, is determined by establishing an equation of motion

$$\ddot{l} = \frac{\cos(\phi - \alpha)}{\cos \phi}(k_{\mathrm{h}} - k_{\mathrm{hy}})g, \tag{13}$$

where g is the earth's gravity. It is noted that k_{hy} is a function of earthquake vertical accelerations that varies throughout the duration of excitation. The permanent displacement along the failure plane is obtained by double-integrating Eq. (13) leading to

$$l = \eta\Delta, \tag{14}$$

where

$$\eta = \frac{\cos(\phi - \alpha)}{\cos \phi}, \tag{15}$$

$$\Delta = \int\int(k_{\mathrm{h}} - k_{\mathrm{hy}})g\,\mathrm{d}t. \tag{16}$$

For convenience of computation, k_{v} may be expressed as a fraction of k_{h}, i.e. $k_{\mathrm{v}} = \lambda k_{\mathrm{h}}$, to avoid using separate set of horizontal and vertical acceleration records. That is, the vertical acceleration is considered to be in phase with the horizontal acceleration. A constant value of k_{hy} is then used in the analysis.

In this study, several real earthquake records are used. The records were scaled to different peak values, k_{ho}, to calculate for permanent displacement. In an actual design, it is advised that past earthquake records from the site or from sites with comparable ground conditions be used. The procedure of numerical integration of these random earthquake records follows that presented in Ling and Leshchinsky [27].

4. Results and discussion

Fig. 2 give the value of N for slopes of different inclinations: $i = 45°$, $60°$, $75°$ and $90°$ and $\phi = 20°$, $30°$ and $40°$. The ratio of vertical to horizontal seismic coefficients is expressed as λ. The results for $\lambda = \pm 0.5$ are shown in the figures. It is seen that N increases with the slope inclination and seismic coefficients. For steep slopes of 75° and 90°, positive λ gives more critical results. However, as the slope flattens, such as $i = 45°$, negative λ gives more critical results when k_{h} becomes large.

It is found that the results obtained for a planar

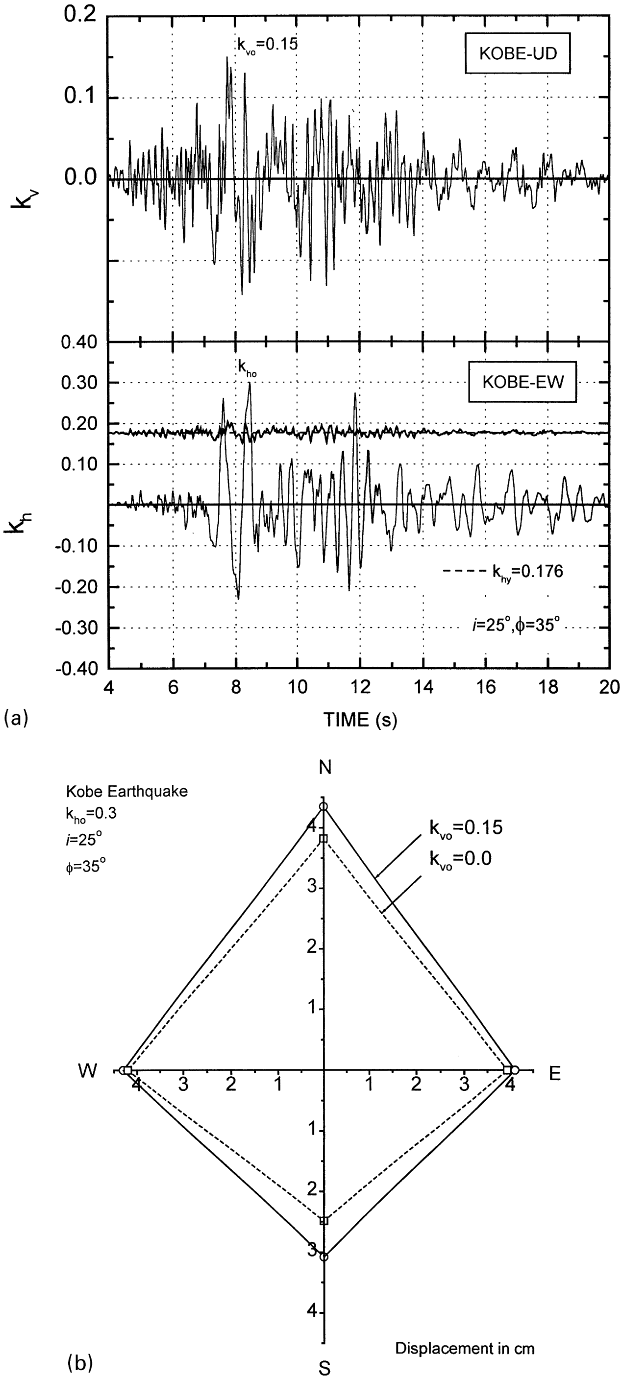

Fig. 5. Example of analysis using Kobe Earthquake records: (a) horizontal, vertical and yield accelerations; and (b) permanent displacements.

mechanism of steep near-vertical slopes are very close to that of log-spiral mechanism [7] (Fig. 3). The difference becomes large as the slope flattens. As formulation based on the planar failure mechanism is straightforward and the solution is computationally less tedious, it is more beneficial for seismic stability analysis of near-vertical slopes.

Under static conditions, the wedge analysis presented herein gives an angle of inclination similar to Coulomb analysis, i.e. $\alpha_{cr} = 45 + \phi/2$. The critical angle of inclination for the M–O analysis was presented by Richards and Elms [31]. A comparison between the wedge analysis and M–O analysis for vertical slope and $\phi = 30°$ (Fig. 4) shows that under seismic acceleration the most critical angle for wedge analysis is steeper than that of M–O analysis. It is also shown that the downward acceleration gives a steeper critical failure plane. Note that a natural slope of 90° having $\phi = 30°$ is not physically possible, but analysis should be for man-made earth structures, such as retaining walls and reinforced soil structures.

Fig. 5(a) shows the acceleration records (EW and UD) for Kobe Earthquake. A slope of inclination $i = 25°$ and $\phi = 35°$ is used as an example. The earthquake records are scaled to a peak horizontal value $k_{ho} = 0.3$ and the peak vertical acceleration $k_{vo} = 0.15$. As k_v varies with time, the yield horizontal acceleration also varies with time as shown by Eq. (12) and in Fig. 5(a). If k_v is considered a constant, taken as k_{vo}, the yield horizontal acceleration is calculated as $k_{hy} = 0.176$. There are a few spikes within the duration of excitation where the yield acceleration is exceeded. The acceleration is then double integrated to obtain permanent displacement. Fig. 5(b) summarises the calculated displacement for the earthquake horizontal acceleration in the directions EW,WE, NS and SN. A slight difference in calculated values is noticed. The results are also included for cases where vertical acceleration is neglected. In most cases, the exclusion of vertical acceleration gives a lower permanent displacement.

The relationships between permanent displacement along the slope, Δ, and excess acceleration, $k_{ho} - k_{hy}$ for Kobe Earthquake records are shown in Fig. 6(a). Δ is multiplied by η to yield l, the permanent displacement along a failure plane. A slight difference between the results calculated from EW and NS records are shown. A comparison between several earthquake records (El-Centro, Imperial records; Taft, Kent County records; Toho-Oki, Tokyo record; Loma Prieta, Capitola record; Northridge, Newhall record; Kobe, JMA EW and NS records) are given in Fig. 6(b) for $k_{hy} = 0.05$ and 0.2. The records from Kobe, Northridge and Toho-Oki give higher displacement than that of El Centro, Taft and Loma Prieta.

Fig. 6(a) can be used conveniently for permanent displacement calculation. For example, for a 25° slope having $\phi = 35°$, under $k_{ho} = 0.3$ and $k_{vo} = 0.0$, the yield acceleration is obtained as $k_{hy} = 0.176$. Fig. 6(a) gives $\Delta = l/\eta = 3.53$ cm. η is calculated as 1.19. Therefore, the permanent displacement is obtained as 4.2 cm.

The curves presented in Fig. 2 would be useful for a permanent displacement limit design of man-made structures, such as reinforced soil structures and waste containment systems. Some aspects of this design are addressed in Ling et al. [32]. In reinforced soil structure design, for example, N presented in Fig. 2 is related to the geosynthetic reinforcement force, and the location of failure surface determines the required geosynthetic length.

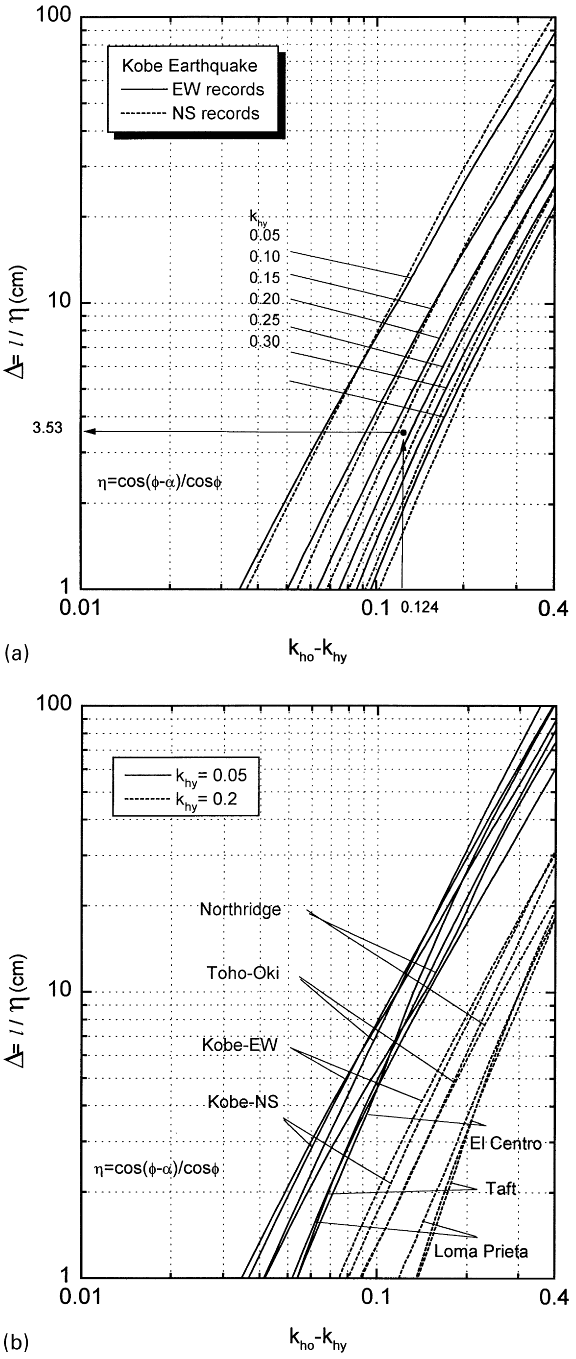

Fig. 6. (a) Relationships between permanent displacement, peak acceleration and yield acceleration for Kobe Earthquake records. (b) Permanent displacements determined from several earthquake records.

5. Conclusions

Francais–Culmann's method of wedge stability analysis is extended to seismic conditions. A vertical component of seismic inertia force, which is typically neglected in current design, was incorporated into stability and permanent displacement analyses. The analysis was demonstrated using real earthquake records. The advantages of extended Francais–Culmann's method lie in the fact that reasonable solutions are obtained for steep slope with simple closed form solution. Same as other failure mechanisms, such as a log-spiral, the simple planar mechanism as presented requires experimental or field verification.

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