

# BOUNDING SURFACE MODEL FOR GEOSYNTHETIC REINFORCEMENTS

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**ABSTRACT:** Geosynthetic reinforcements are manufactured from polymers such as polyester, polypropylene, and high-density polyethylene. Compared to metals, they exhibit large plastic strains, and highly nonlinear and hysteretic behaviors under cyclic loading. Most geosynthetic reinforcements do not sustain compression load. Traditional cyclic models with the Masing rule fail to simulate such unique behavior. A 1D bounding surface concept is used to develop a model that considers these unique properties of geogrids. The unique features of the model include nonparallel bounding lines and different loading and unloading hardening parameters. The model was calibrated and compared with the experimental results of two geogrids under monotonic and cyclic loadings with different load amplitudes.

## INTRODUCTION

Examples of models that are frequently used to simulate the cyclic loading of structural members include the bilinear model, Ramberg-Osgood and hyperbolic models combined with the Masing rule, and plasticity models considering isotropic hardening. They are inadequate for simulating the complicated loading paths of the structural members during seismic events, especially with reference to the Bauschinger effect. The models with a kinematic hardening rule can be more suitable for considering the Bauschinger effect, but numerical implementation to include the change of the strain hardening modulus and yield stress under load reversal is difficult. A review of different models is given by Kato (1979) and Crisfield (1997). The multisurface model has also been developed to simulate the cyclic behavior of materials (Mroz 1969). However, this model has the disadvantage of employing piecewise linear stress-strain relationships with a complicated computation scheme because of the numerous yield surfaces involved under cyclic loading.

The bounding surface model, which was developed by Dafalias and Popov (1975, 1976, 1977) and independently by Krieg (1975), better simulated the stress-strain hysteresis under cyclic loading. The model may be considered as a simplified version of the multisurface model. In the bounding surface model (Fig. 1), for typical stress-plastic strain ( $\sigma$ - $\epsilon_p$ ) relationships, the current state of stress  $\sigma$  is related to the image stress  $\bar{\sigma}$  at the boundary surfaces  $XX'$  and  $YY'$  through a mapping rule ( $\bar{\sigma}_+$  for continued loading and  $\bar{\sigma}_-$  for reversed loading). The plastic modulus  $E_p$  is obtained as a function of the distance between  $\bar{\sigma}$  and  $\sigma$ , denoted as  $\delta$ , and the slope of the bound at  $\bar{\sigma}$ , denoted as  $\bar{E}_p$ . That is

$$E_p = \frac{d\sigma}{d\epsilon_p} = \bar{E}_p + h \frac{\delta}{\delta_{in} - \delta} \quad (1)$$

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where  $\delta_{in}$  value of  $\delta$  at the start of yielding along the current loading path; and  $h$  = hardening shape parameter that is related to  $\delta_{in}$ . Note that  $\delta$  and  $h$  are both positive, and  $\delta_{in}$  acts as a discrete memory parameter of the most recent event of unloading-reloading. If the bounds remain straight parallel lines then  $\bar{E}_p = \bar{E}_p^o = \text{constant}$ . Generally,  $\bar{E}_p$  varies with the hardening of the bounds.

One of the features of the bounding surface model is that plastic strain is allowed inside the bounding surface, and thus is very relevant for simulating cyclic behavior of any material. There is also a smooth transition between the elastic and plastic states. Different versions of the bounding surface model, under uniaxial conditions, have been proposed for steel elements [e.g., Pettersson and Popov (1977), Tseng and Lee (1983), Cofie and Krawinkler (1985), and Dafalias (1992)]. Recently, a generalized version of the bounding surface model was proposed by Shen et al. (1995) that included special considerations to the yield plateau, moving bounding lines, and the slope of the bounding lines.

In this technical note, the development of a 1D bounding surface model that simulates the cyclic behavior of major types of geogrids is described. The validation of the proposed model and the calibration procedures are discussed.

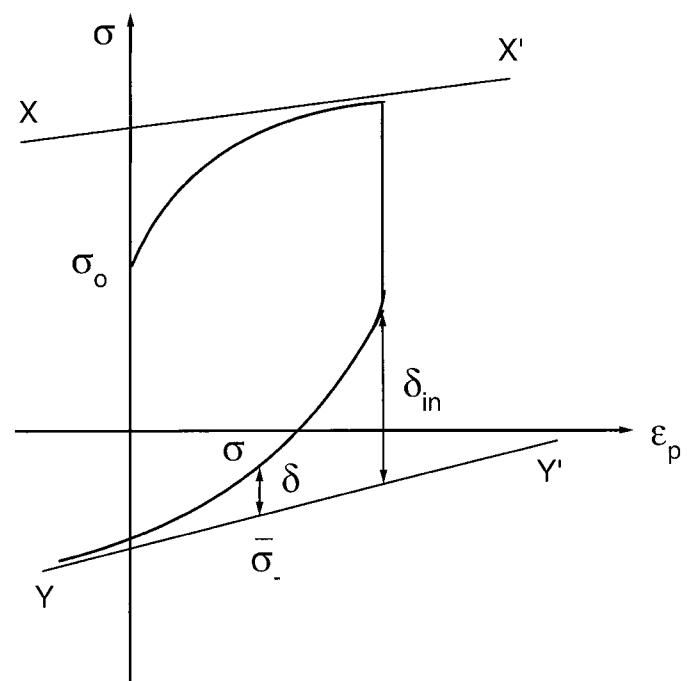


FIG. 1. Conceptual Sketch of Bounding Surface Model

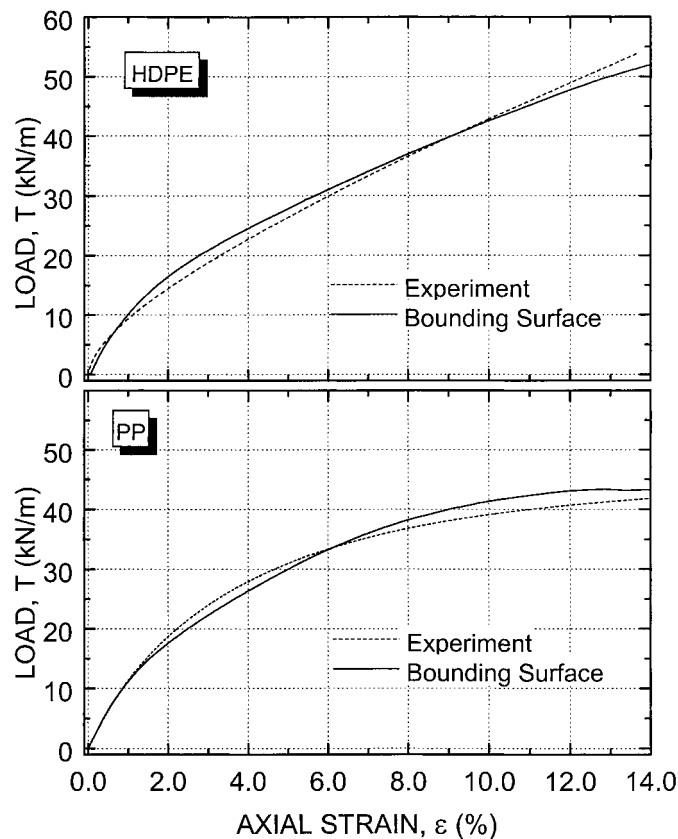
## BOUNDING SURFACE MODEL FOR GEOGRIDS

Geosynthetic materials have been used as tensile reinforcement in many soil structures such as slopes, retaining walls, roadways, and waste containment systems. They are manufactured from polymers such as polyester, polypropylene, and high-density polyethylene (Koerner 1998). The tensile properties of geosynthetic materials are usually expressed using load-strain ( $T$ - $\epsilon$ ) relationships, with the stiffness  $J$  obtained from the slope of the curve.

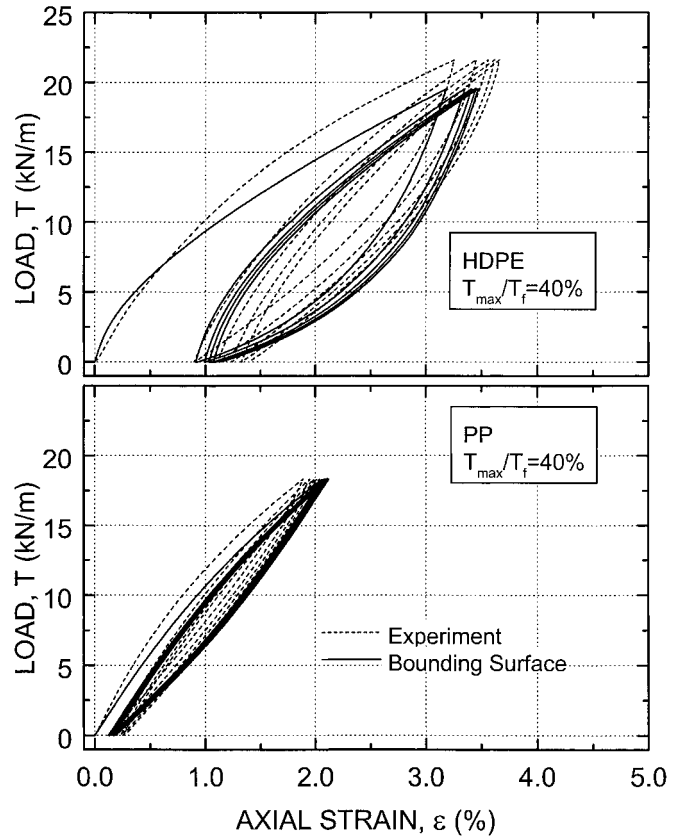
The behavior of reinforced soil structures under seismic loading has received considerable attention [e.g., Ling and Leshchinsky (1998)], and it has been reported that soil structures reinforced with polymeric geosynthetic materials gave excellent performance during earthquakes (Tatsuoka et al. 1998). Several experimental studies have been conducted on the cyclic behavior of geogrids [e.g., Bathurst and Cai (1994), Ashwamy and Bourdeau (1996), and Ling et al. (1998)]. The geosynthetic materials are modeled with linear elastic properties for earthquake loading. Yogendrakumar et al. (1992) has

**TABLE 1.** Parameters for Bounding Surface Model of Geogrid

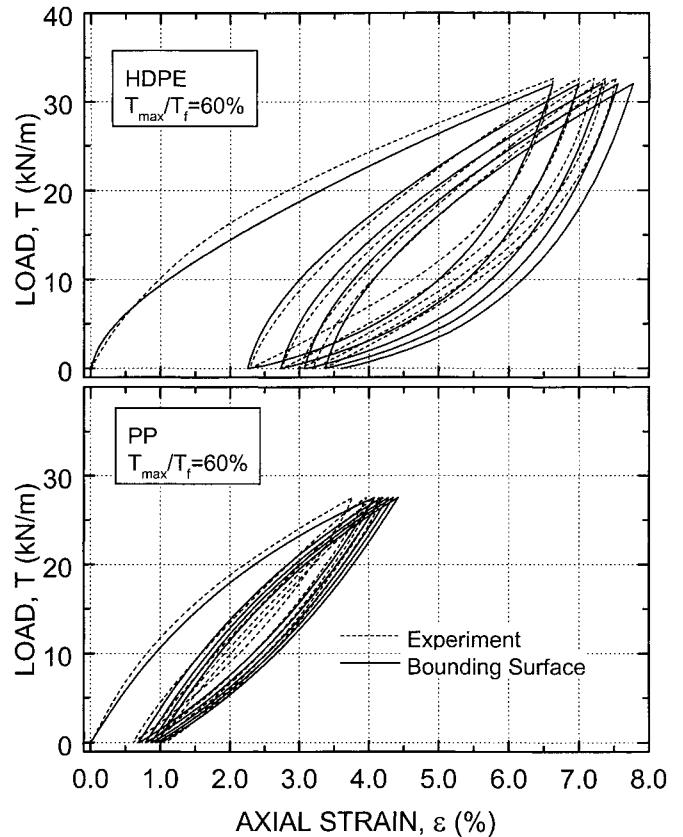
Parameter	Polypropylene (kN/m)	Polyethylene (kN/m)
$J_e$	1,300.0	2,500.0
$A$	8.0	281.0
$B$	-20.0	-40.0
$J_{p+}^o$	43.5	28.5
$J_{p-}^o$	-15.0	-4.0
$h_o^L$	700.0	150.0
$h_o^U$	1,200.0	1,200.0
$h_k^L$	3,500.0	400.0
$h_k^U$	1.0	0.0



**FIG. 2.** Comparison between Bounding Surface Model and Experimental Results for Monotonic Loading



**FIG. 3.** Comparison between Bounding Surface Model and Experimental Results for Cyclic Loading: Load Level = 40%



**FIG. 4.** Comparison between Bounding Surface Model and Experimental Results for Cyclic Loading: Load Level = 60%

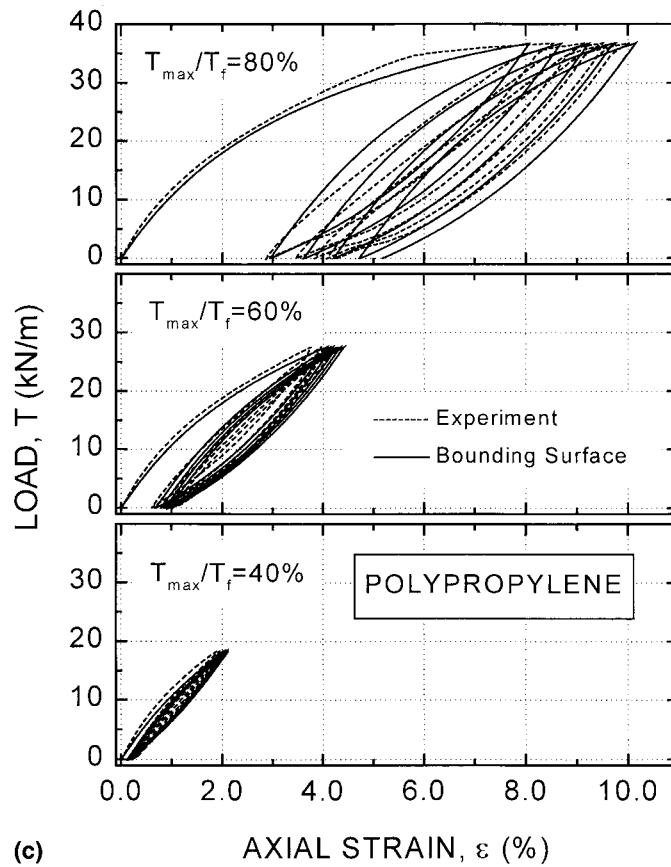
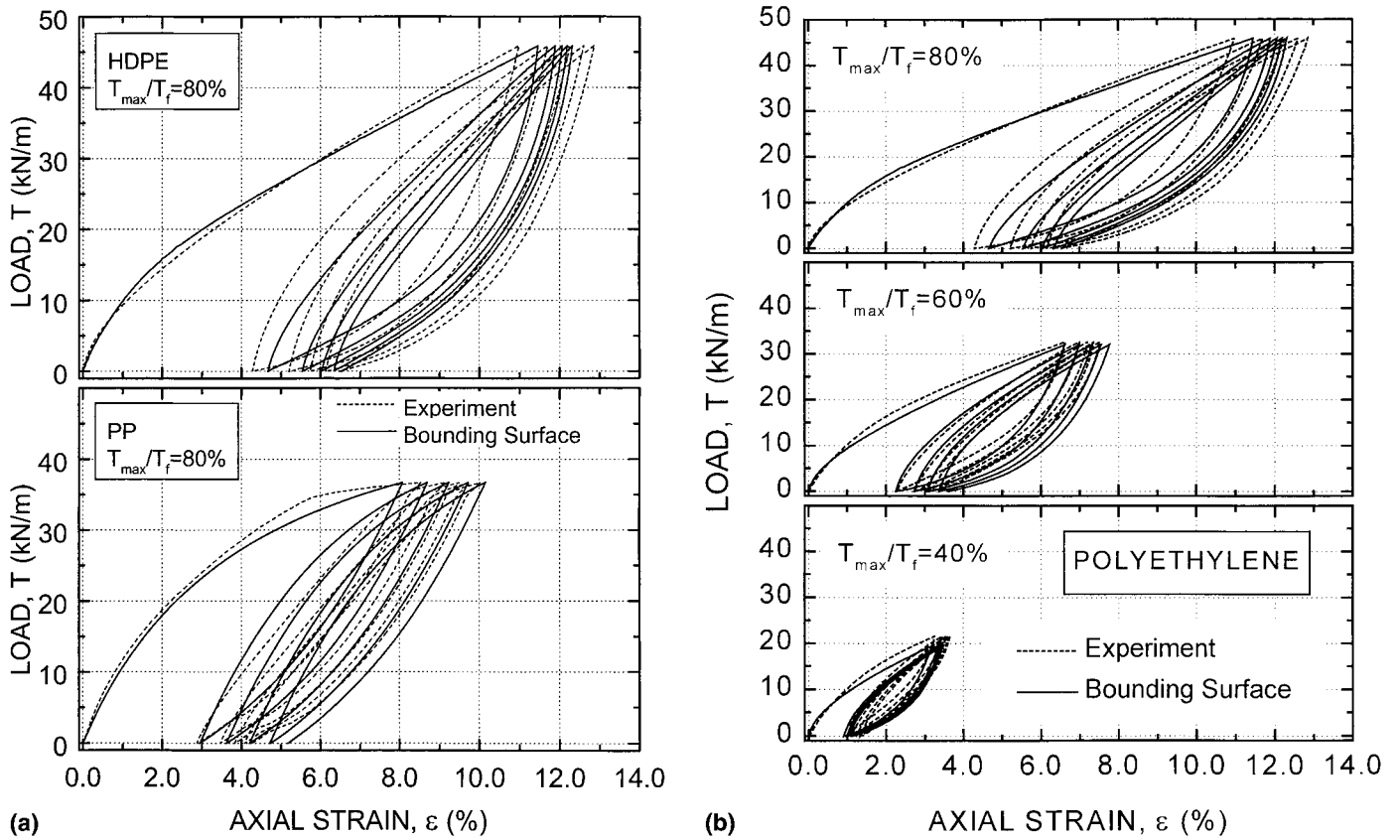


FIG. 5. Comparison between Bounding Surface Model and Experimental Results for Cyclic Loading: Load Level = 80%

developed a cyclic model for a geogrid based on the hyperbolic relationships combined with the Masing rule. Note that the model was used for an equivalent linear analysis.

Geosynthetic materials exhibit nonlinear stress-strain relationships compared to steel and other metallic reinforcements. Under cyclic loading, the hysteresis and damping are more significant compared to metallic reinforcements. Moreover, geogrid reinforcements do not resist compression and develop large plastic strain during cyclic loading (Ling et al. 1998). The main features of the proposed 1D model that accommodates the aforementioned characteristics of geogrids are as follows.

### Bounding Lines

In the proposed model, since geogrids do not sustain compression, the bounding line  $YY'$  in the compression side of the load-plastic strain ( $T-\epsilon_p$ ) curve, which is used to express the unloading curve, is not parallel to line  $XX'$  of the tension side. The bounding lines on the tension and compression sides of the  $T-\epsilon_p$  curve are expressed using straight lines. Thus, a total of four parameters are used to define the bounding lines

$$T_+ = A + J_{p+}^o \epsilon_p; \quad T_- = B + J_{p-}^o \epsilon_p \quad (2a,b)$$

where  $J_{p+}^o$  and  $J_{p-}^o$  = values of stiffness or slopes of the bounding lines; and  $A$  and  $B$  = intercepts at the  $T$ -axis on the tension and compression sides of the load-strain curve, respectively.

### Hardening Parameters

A single hardening parameter  $h$  is inadequate for simulating the cyclic behavior of geogrids. The hardening is controlled by the plastic strain of the geogrids, but it is different for loading and unloading curves. Moreover, the hardening properties for reloading curves are different from a monotonic loading curve. In this proposed model, different hardening parameters are defined for the unloading and reloading curves through the power functions

$$h^L = h_o^L + h_k^L \epsilon_p^{\alpha_L}; \quad h^U = h_o^U + h_k^U \epsilon_p^{\alpha_U} \quad (3a,b)$$

where  $h_o^L$ ,  $h_o^U$ ,  $h_k^L$ ,  $h_k^U$ ,  $\alpha_L$ , and  $\alpha_U$  = constants.

### Total Stiffness

Rewriting (1), the plastic stiffness of the geogrid is obtained as

$$J_p = \frac{dT}{d\epsilon_p} = \bar{J}_p + h \frac{\delta}{\delta_m - \delta} \quad (4)$$

where  $\bar{J}_p$  = slope of the bounding line. The total stiffness that relates load and strain of the geogrid is obtained as

$$J_T = \frac{J_p J_e}{J_p + J_e} \quad (5)$$

where  $J_T$ ,  $J_e$ , and  $J_p$  = total, elastic, and plastic stiffness, respectively. In the elastic range, before yielding is exceeded, the total stiffness  $J_T$  is set equal to the elastic stiffness  $J_e$  of the geogrid.

### MODEL CALIBRATION

The proposed bounding surface model is verified with the experimental results obtained for two geogrids: one manufactured from polypropylene (PP) and the other from high-density polyethylene (HDPE). The properties of geogrids and test conditions are given in Ling et al. (1998). The uniaxial tests were conducted on a single rib specimen, 22 and 33 cm, respectively, for the PP and HDPE geogrids. A monotonic test was conducted at a constant strain rate of 10%/min to obtain the

peak strength  $T_f$ . Then, cyclic tests were conducted with a load amplitude  $T_{max}$  at 10, 20, 40, 60, and 80% of the peak strength. The strain rate of cyclic tests was 10%/min for both loading and unloading, with a total of 100 cycles.

The best-fit parameters were sought for the geogrids at all load levels. The yielding load was given a minimum value of 1.0 kN/m to facilitate calculation at the initial stage of loading, since the geogrids exhibit primarily plastic strains. It is seen that the values of hardening parameters  $\alpha_L$  and  $\alpha_U$  are the same for both geogrids (including another polyester geogrid that is not reported herein). Consequently, (3a) and (3b) may be rewritten as

$$h^L = h_o^L + h_k^L \sqrt{\epsilon_p}; \quad h^U = h_o^U + h_k^U / \epsilon_p \quad (6a,b)$$

Thus, a total of nine parameters, excluding the default yielding load, are assigned for the proposed bounding surface model. Table 1 lists the values of these parameters for the two geogrids. Note that the virtual bounding line has a negative slope in responding to the noncompression behavior of the geogrids. In the case of monotonic loading, five parameters ( $J_e$ ,  $A$ ,  $J_{p+}^o$ ,  $h_o^L$ , and  $h_k^L$ ) are required to define the model.

Fig. 2 shows the comparison between the results obtained from the experiments and simulation by the bounding surface model for the two geogrids under monotonic loading conditions. In the cyclic loading, several cycles were simulated. Figs. 3–5 show the comparison between the test results and the simulation. Note the difference in scales used in each figure, and the fact that a single set of parameters was used for each geogrid over different load levels. It is seen that the model is capable of replicating the monotonic loading curve as well as the loading-unloading curves for the two geogrids.

### CONCLUSIONS

The bounding surface concept has been used to develop a 1D geogrid model that simulates highly nonlinear stress-strain behavior and hysteresis under cyclic loading. The model has nonparallel bounding lines to accommodate for the noncompressional behavior of the geogrids. The model favorably simulated the experimental results of two geogrids tested under monotonic loading and cyclic loading at different load amplitudes.

Currently, the model requires nine material parameters for cyclic loading, and five parameters in the case of monotonic loading. However, the total number is expected to reduce following additional calibration studies.

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